The Performance Analysis and Realization of the Digital LLRF Feedback Control System for PEFP Copper Model of Superconducting Linac**

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1. Introduction

In Proton Engineering Frontier Project (PEFP), a digital I/Q feedback control loop which based on the FPGA technique, will be used in the superconducting linac's LLRF system [1]. This feedback loop was mainly for suppressing amplitude and phase fluctuation of the RF field inside of accelerating cavity. Due to time delay in the control loop, the system performance analysis to keep system working stable was necessary. To obtain sufficient noise suppression, which results from RF field disturbance, a care for loop parameters selection was required. For given magnitude margin and phase margin, the Bode diagram, the step response, and the PI controller parameters were obtained. According to the analysis, a special experiment setup of the digital LLRF control system was constructed. With proper parameter select, the close loop lock-in was realized.

2. The performance analysis of the control system for the digital I/Q LLRF feedback control loop

The control system illustrated as Fig.1. [1] It is mainly a PI controller. Due to I and Q components of the IF signal have the same loop parameters, here, only one loop was shown in the diagram.

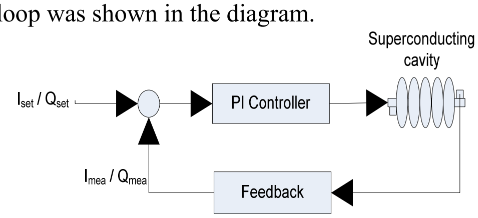

Fig. 1 Block diagram of a LLRF closed-loop control system

The PI has the transfer function $G_{PI}(s) = K_p\left(1+\frac{K_i}{s}\right)$, here K_p, K_i are the proportional and integral coefficients respectively. Superconducting cavity has the transfer function $G_{plant}(s) = \frac{\omega_{1/2}}{s+\omega_{1/2}}$ here $\omega_{1/2}$ is its half of 3 dB bandwidth. The feedback loop has total time delay which was expressed as $G_\tau(s) = e^{-\tau s}$. [2, 3] For simplicity in practice, we always select $K_i = \omega_{1/2}$, i.e. the integral coefficient is equal to the bandwidth of the cavity, and the open-loop transfer function has the magnitude-frequency response and phase-frequency response which are

$$\left|G(j\omega)\right| = \left|\frac{K_p\omega_{1/2}}{j\omega}e^{-j\omega\tau}\right| = \frac{K_p\omega_{1/2}}{\omega} \quad (1)$$

$$\angle G(j\omega) = -\tau\omega - \frac{\pi}{2} \quad (2)$$

From (2), the *phase crossover frequency* can be found as $\omega_p = \frac{\pi}{2\tau}$. By substituting it into equation (1), the magnitude-frequency response becomes as

$$\left|G(j\omega)\right| = \frac{K_p\omega_{1/2}}{\omega_p} = \frac{2\tau K_p\omega_{1/2}}{\pi} \quad (3)$$

In a practical control system, enough magnitude margin and phase margin are necessary. From the knowledge of modern control theory [2, 3], the *magnitude margin* should be roughly larger than *6 dB*, and the *phase margin* should be roughly larger than 45°. With 6db *magnitude margin*, from the expression (3), the proportional gain coefficient could be obtained

$$K_{p-6dB} = \frac{\pi}{4\tau\omega_{1/2}} = \frac{1}{8\tau f_{1/2}} \quad (4)$$

For the test result of superconducting cavity in PEFP, $\tau = 5\mu s$, $f_{1/2} = 23kHz$ (copper cavity in room temperature), $\omega_{1/2} = f_{1/2}/2\pi = 145krad/s$, therefore $K_{p-6dB} \le 1.087$ could be obtained.

3. The experimental parameter settings

The phase margin is defined as the extra loop delay to obtain **critical stability** at the *gain crossover frequency* ω_g. With the equation (3), let $\left|G(j\omega)\right| = \frac{K_p\omega_{1/2}}{\omega} = 1$, replace K_p with above equation (4), and resolve the equation for ω, we obtain the *gain crossover frequency* which is $\omega_g = \frac{\pi}{4\tau}$, by substituting it into (2), the phase angle becomes as $\angle G(j\omega_g) = -\tau\omega_g - \frac{\pi}{2} = -\frac{3\pi}{4}$.

The *phase margin* of system is

$$\gamma = \pi + \angle G(j\omega_g) = \pi - \frac{3\pi}{4} = \frac{\pi}{4} \qquad (5)$$

Fig. 2 is the Bode diagram when magnitude margin is 6 dB, phase margin is 45º. Fig. 3 shows the step response in the close-loop at the same magnitude margin and phase margin. Both plots are using MATLAB.

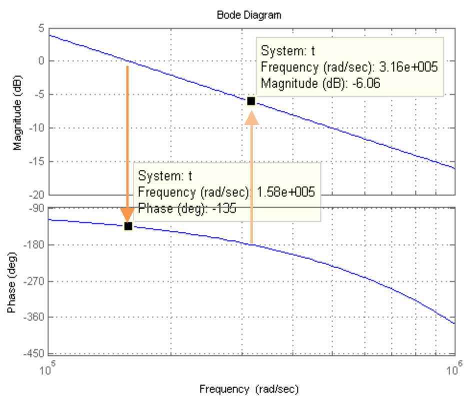

Fig.2 Bode diagram for 6 dB Magnitude Margin

From Fig. 2 we can see, at frequency 50 KHz, phase -180º, magnitude -6dB; at frequency 25 KHz, phase -135º, magnitude 0 dB.

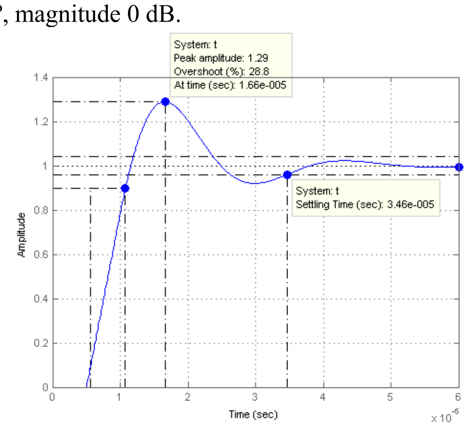

Fig. 3 Step response for 6 dB Magnitude Margin

Form Fig.3 we can see the step response has an overshoot of 28.8% at the time of 16.6 μs, the rise time of lead edge was 5 μs, the settling time is 35 μs and the time to achieve steady state was 60 μs.

The PI Controller Parameters for 6dB magnitude margin and 45° phase margin could be summarized as follow Table 1.

Table 1: The practice PI controller parameter settings

Time delay	τ	5 μs
Cavity bandwidth	$\omega_{1/2}$	145 000 Rad/s
	$f_{1/2}$	23 KHz
Phase crossover frequency	$\omega_p=\pi/2\tau$	314 000 Rad/s
	$f_p=1/4\tau$	50 KHz
Gain crossover frequency	$\omega_g=\pi/4\tau$	157 000 Rad/s
	$f_g=1/8\tau$	25 KHz
Proportional gain	$Kp=1/8\tau f_{1/2}$	1.087
Integratal gain (1)	$Ki=\omega_{1/2}$	145 000
Integratal gain (2)	$Ki=K_p*K_i$	157 000

From the Table 1, we can know that if the system settings choose the $\mathbf{K_p}$ **< 1.087** and $\mathbf{K_i=\omega_{1/2}}$, thus it could meet with the requirement of the **stability**.

4. The realization of the experiment and results

We consider above calculation with the experiment. For the stability requirement, the open-loop has the gain about 0.035. If we use zero cancel pole scheme and 6 dB magnitude margin, the experimental P-gain settings is 1.087/0.035=31, I-gain settings is 185k/0.035=4.5*10^6 for system stability.

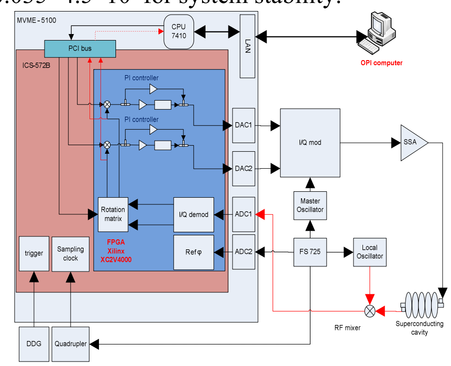

Fig. 4 The experimental system setup

In experiment, we use P-gain about 15, I-gain about 3*10^6, and open-loop gain about 0.018. The second time we try P-gain 20, I-gain about 4*10^6. Both times the system can locked-in. If we change P-gain to 50, others are not changed; the system can't locked-in. By using an experimental system setup shown in Fig.4, a 3-hours long-term stability operation test was accomplished. The long-term stablity test has been proved that the system could be locked-in.

5. Conclusion

Through experiment, the stability analysis about the choices, especially for range of $K_{p\text{-}6dB}$, K_i gain parameters of the Proportional Integral (PI) controller were given. We tried a P-gain within this range and obtained a good close-loop performance. We tried a P-gain value far beyond the range. In this case, it can't control in the close-loop, that means the system doesn't locked-in.

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